

Pendry Replies: The preceding Comment [1] takes exception to the multiple scattering calculation made in my original Letter [2] on the perfect lens, though the conclusions of that calculation are accepted as correct by 't Hooft. In this Reply I explain that the original calculations are in accord with multiple scattering theory as documented in standard textbooks.

The formula in dispute describes transmission into a slab of material and out the other side:

\[
T_S = t' \exp(ik'z d) + t' r'^2 \exp(3ik'z d) + t' r'^4 \exp(5ik'z d) + \ldots \\
= \frac{t' \exp(ik'z d)}{1 - r'^2 \exp(2ik'z d)},
\]

(1)

where \( t \) and \( t' \) are the transmission coefficients of the entrance and exit surfaces of the slab, \( r' \) is the reflection coefficient of the internal surface of the slab, and

\[
\exp(ik'z d)
\]

(2)

is the phase factor acquired when waves cross a slab of material, thickness \( d \). 't Hooft has several objections which I answer as follows.

The first objection is to the causal boundary conditions I refer to in my Letter which are used to calculate \( t, t' \), and \( r' \). These are conventional in scattering theory of any sort of particle and a complete discussion of multiple scattering of electrons can be found in [3]. Causal boundary conditions require that a wave incident on a boundary must be matched to waves on the far side of the boundary which either carry flux away from that boundary or which decay exponentially away from the boundary. In the presence of absorption these statements are equivalent because in a causal medium waves always decay in the direction of the current. Any waves that grow in amplitude away from the boundary can be generated only by reflection at the second interface and by other higher order multiple scattering processes. The simple calculation proposed by 't Hooft works only at the frequency where

\[
\varepsilon(\omega) = -1, \quad \mu(\omega) = -1
\]

(3)

but will not otherwise give a correct answer.

The second objection is that \( T_S \), the transmission coefficient in (1), can exponentially increase with thickness of the slab. Since multiple scattering generates both a growing and a decaying wave inside the slab, it is only a question of which predominates as to whether the wave has a net amplification or decay as it crosses the slab. Both cases are physically allowable.

The third objection is that the series diverges in the case to which I apply it. Summation of an infinite series is a standard technique in multiple scattering theory. Provided that the summation is made exactly, the correctness of the answer survives the formal divergence of the series, a result that can be understood through arguments of analytic continuation. The topic is dealt with in Ref. [3] and more completely in [4].

A final point in the Comment concerns energy transport, which is alleged to create problems for the theory. This is not the case. In an ideal situation in the steady state and where there is no absorption of light in the system, a purely decaying wave emerges from the slab into vacuum. Such a wave transports no energy and, therefore, as stated in the original Letter, energy conservation is not violated by amplification. 't Hooft raises the question of what happens if an absorber is placed in the detector plane. The answer is the following: the absorber will reflect the incident wave so that we now have in the space between the slab and the absorber two waves decaying in opposite directions. Provided that there is a difference in phase between these waves, energy can flow. These are effects that can be observed without recourse to negative refraction and can occur in any tunneling experiment for electrons or photons. Likewise the transients present when the fields are initially switched on can be calculated by evaluating the transmission formula (1) as a function of frequency, and Fourier transforming. It must be borne in mind that if \( \varepsilon \) and \( \mu \) take negative values they must inevitably disperse strongly with frequency. The perfect lens condition can be satisfied only at a single frequency.

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