All-angle negative refraction without negative effective index

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We describe an all-angle negative refraction effect that does not employ a negative effective index of refraction and involves photonic crystals. A few simple criteria sufficient to achieve this behavior are presented. To illustrate this phenomenon, a microsuperlens is designed and numerically demonstrated.

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Negative refraction of electromagnetic waves in “left-handed materials” has become of interest recently because it is the foundation for a variety of novel phenomena.1–6 In particular, it has been suggested that negative refraction leads to a superlensing effect that can potentially overcome the diffraction limit inherent in conventional lenses.2 These phenomena have been described in the context of an effective-medium theory with negative index of refraction, and at the moment only appear possible in the microwave regime. To explore the possibility of negative refraction in the optical regime, one may turn to photonic crystals as interesting alternatives. Recent experimental7 and theoretical8 works indicate that negative refraction phenomena in photonic crystals are possible in regimes of negative group velocity and negative effective index above the first band near the Brillouin-zone center (Γ). However, lower frequencies in the band structure may be more desirable in high-resolution superlensing, as we discuss later in this paper. Here, we show that negative refraction can also be achieved without employing materials with negative effective index. In particular, our focus is on the lowest photonic band near a Brillouin-zone corner farthest from Γ. Interestingly, this band has a positive group velocity and a positive refractive index, but a negative photonic “effective mass.” We exhibit a frequency range so that for all incident angles one obtains only a single, negative-refracted beam. Such all-angle negative refraction (AANR) is essential for superlens applications.

Although our analysis is general, we study two dimensional (2D) photonic crystals for simplicity. We begin with TE modes (in-plane electric field) and consider a square lattice of air holes in dielectric ε = 12.0 (e.g., Si at 1.55 μm), with lattice constant a and hole radius r = 0.35a. To visualize and analyze diffraction effects, we employ wave-vector diagrams: constant-frequency contours in k space whose gradient vectors give the group velocities of the photonic modes. Our numerical calculations are carried out in a plane-wave basis by preconditioned conjugate-gradient minimization of the block Rayleigh quotient using a freely available software package developed in-house.9 A root finder is used to solve for the exact wave vectors that lead to a given frequency. The results for frequencies throughout the lowest photonic band are shown in Fig. 1.

We observe from Fig. 1 that due to the negative-definite photonic effective mass ∂2Ω/∂k∂k, at the M point, the frequency contours are convex in the vicinity of M and have inward-pointing group velocities. For frequencies that correspond to all-convex contours, negative refraction occurs as illustrated in Fig. 2. The distinct refracted propagating modes are determined by the conservation of the frequency and the wave-vector component parallel to the air/photonic-crystal surface. If the surface normal is along ΓM [(11) direction], and the contour is everywhere convex, then an incoming plane wave from air will couple to a single mode that propagates into this crystal on the negative side of the boundary normal. We have thus realized negative refraction in the first band.

It is clear from this example that neither a negative group velocity nor a negative effective index is a prerequisite for negative refraction. In fact, the lowest band here has k·∂Ω/∂k≥0 everywhere within the first Brillouin zone, meaning that the group velocity is never opposite to the phase velocity. In this sense, we are operating in a regime of positive effective index. In fact, our photonic crystal is behaving much like a uniform, “right-handed” medium with hyperbolic dispersion relations, such as those induced by an-

FIG. 1. (Color) Several constant-frequency contours of the first band of a model photonic crystal, drawn in the repeated zone scheme. Frequency values are in units of 2πc/a.
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To realize AANR for superlensing, the required conditions in our model system are that the photonic-crystal contours be both convex and larger than the constant-frequency contours for air, i.e., circles with radius \( \omega/c \). Incident beams at any incident angle will then experience negative refraction when entering the photonic crystal. Note that single-beam behavior is only possible for \( \omega \approx 0.5 \times 2 \pi c/a_i \) (where \( a_i \) is the surface-parallel period) in order to avoid diffraction. Thus, there are three key criteria that are sufficient to guarantee single-beam AANR.

(i) The constant-frequency contour of the photonic crystal is all convex with a negative photonic effective mass.

(ii) All incoming wave vectors at such a frequency are included within the constant-frequency contour of the photonic crystal.

(iii) The frequency is below \( 0.5 \times 2 \pi c/a_i \).

Using these criteria, we have calculated the AANR frequency range in our model system. We find two regions where AANR is possible, as shown in Fig. 3. These exist in the first and second bands, and correspond to positive and negative effective indices, respectively. The lower-frequency range has an upper limit \( \omega_p = 0.198 \times 2 \pi c/a_i \) that is obtained directly from the band structure by finding the intersection with the light line as depicted in Fig. 3. We find the lower limit to be \( \omega_l = 0.186 \times 2 \pi c/a_i \), by computing the frequency at which the radius of curvature of the contours along \( \Gamma M \) diverges. This leads to a fractional AANR frequency range of 6.1% around 0.192 \( \times 2 \pi c/a_i \). For the second band, we obtain an AANR range of only 0.7% around 0.287 \( \times 2 \pi c/a_i \).

To demonstrate how AANR can be put to use, we design a superlens using the same photonic crystal. Ideally, such a superlens can focus a point source on one side of the lens into a real point image on the other side even for the case of a parallel-sided slab of material.\(^1\)\(^2\) Such a superlens possesses several key advantages over conventional lenses. Due to the lack of an optical axis, strict alignment is not necessary. Moreover, flat slabs instead of curved shapes are used and thus fabrication may be easier in principle. A superlens also operates over distances on the order of wavelengths and is an ideal candidate for small-scale integration. Furthermore, AANR for a superlens means that there is essentially no physical limit on the aperture of this imaging system. Finally, for superlensing at a given configuration and wavelength \( \lambda \), the resolution of a superlens is expected to be limited by the surface period \( a_i \), the characteristic length in our problem. Thus the frequency region of preference, yielding the potentially highest resolution, should correspond to the smallest \( a_i/\lambda \). Since typically \( a_i \sim a \) and \( a/\lambda = a/2\pi c \), this implies that we should choose to operate at the lowest AANR values of \( a/2\pi c \) in our band structure.

In order to model such a superlens, we have performed finite-difference time-domain (FDTD) simulations with perfectly matched layer boundary conditions\(^10\) on a parallel-sided (11)-oriented slab of our photonic crystal as shown in Fig. 4. Depicted here is a snapshot of the magnetic field for a continuous-wave (CW) point source placed at a distance 0.35\( a \) from the left-hand surface. The frequency is 0.195 \( \times 2 \pi c/a \), chosen to lie within the lowest AANR frequency range. Note the formation of a “point” image on the right-hand side of the superlens at a distance of 0.38\( a \). Moreover,

![FIG. 2. (Color) Left panel: negative-refracted beams constructed from constant-frequency contours and conservation of surface-parallel wave vector. Thick arrows indicate group-velocity directions, and thin arrows stand for phase-velocity directions. Right panel: diagram of refracted rays in the actual crystal.](image)

![FIG. 3. (Color) The AANR frequency ranges are highlighted in red in the band structure. The light line shifted to \( M \) is shown in blue.](image)
even though $a_s = \sqrt{2}a$ in this case, the frequency is low enough that we obtain a transverse size of only $0.67\lambda$. Although small aberrations are visible in the field pattern, the simulation clearly demonstrates the superlensing effect of this photonic crystal. A similar calculation in the second band AANR region for a slab oriented along $(10)$ with $a_s = a$ also gives a focused image. Even though $a_s/a$ is smaller now, the image turns out to have a larger transverse size $0.76\lambda$, in accordance with our intuition that lower frequencies in the band structure offer better superlensing resolution. For both cases the thickness and surface termination of our slab in the simulation are chosen to minimize reflections.
This is accomplished by requiring a slab to possess both mirror symmetry and a thickness equal to an integer multiple of half the wavelength in the slab. Normal-incidence transmission through the slab then reaches a resonance maximum for the source frequency and stays above 99% for a range of incident angles of typically at least ±40°. The slab thickness also determines the maximum object distance from the left-hand face that can lead to a good image at a given frequency: the ray-crossing point induced by negative refraction must lie within the superlens. In general, thicker slabs will be able to focus more distant objects.

The preceding discussion focused on the TE modes of a “holes-in-dielectric” structure. However, based on the general criteria presented here one can obtain single-beam AANR for TM modes in a similar holes-in-dielectric system with dielectric-rod radii

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Although we have focused our analysis on 2D photonic crystals, it should be noted that such 2D-periodic crystals can be readily studied in 3D. For example, the experiments of McCall et al. use a photonic-crystal slab sandwiched between two metal plates and exhibit 2D TM modes in the microwave regime. We conjecture that similar results might also be obtainable in the optical regime by replacing the metallic components with multilayer films with a large gap, or simply by index guiding. Although true single-beam refraction is difficult to achieve in 3D, our ideas of AANR with positive effective index also generalize naturally to 3D-periodic crystals and could lead to realistic superlens applications.

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