Unidirectional light propagation at exceptional points

Xiaobo Yin and Xiang Zhang

Unique opportunities arise from exceptional points that coalesce states of an open system in synthetic photonic media, where delicately balanced complex dielectric functions produce unprecedented optical properties.

The magic garment worn by the fictional wizard Harry Potter makes him invisible and simultaneously allows him to see the outside world. The concept of this type of invisibility cloak has captured the imagination for centuries and has inspired many recent explorations in the field of optics. Making such a fictional garment is indeed a grand challenge: when invisible to the outside observer, the concealed individual cannot see through the present-day cloaks due to the reciprocity of light. The present cloaking schemes hide an object by redirecting the flow of light smoothly around it without reflection and scattering, much like water flowing around a rock in a stream, rendering the object invisible to the downstream observer[1–3]. To realize the magic Harry Potter cloak, however, one has to break the reciprocity of light — a seemingly impossible task. Recently, manipulating parity-time symmetry in a synthetic photonic medium with a judicious arrangement of the dielectric constants of the constituent materials has led to intriguing optical phenomena, including the one-way reflectionless propagation of light and the 'unidirectional invisibility'[4–6]. But can it cast new light on accomplishing the fictional garment worn by Harry?

By designing the spatial distribution of materials, extraordinary electromagnetic properties may be obtained. An example is a photonic crystal, which is typically created by periodically arranged non-absorbing dielectrics at the wavelength scale. Photonic crystals not only allow for manipulating the propagation of light owing to the existence of 'forbidden-gaps', but also enable controlled light–matter interactions[7,8]. Introducing a dispersive and non-perturbative imaginary part to the dielectric constant profile, however, destroys the mode orthogonality across different bands, leaving the system open and sensitive to the complex dynamics of gain and/or loss[9,10]. Parity-time synthetic matter, on the other hand, modulates the materials' refractive index, gain and loss in a strong but delicate and balanced manner (Fig. 1). This leads to interesting optical phenomena and devices, such as asymmetric power oscillations between two waveguides and unidirectional reflectionless light propagation[4–6,11–14].

> When invisible to the outside observer, the concealed individual cannot see through the present-day cloaks due to the reciprocity of light.

The parity-time symmetry of an open system has been a useful notation in the fields of complex algebras[15] and quantum-field theories[16], and has recently been introduced in optics[4–6,11–14]. To conserve the parity-time symmetry, the system has to be invariant under the combined operations of time-reversal and parity inversion, resulting in entirely real-energy spectra[17]. In optics, such operations can be realized by a spatial arrangement of the refractive index with a gain and/or loss profile in a composite medium. For instance, one can modulate the real component of the indices (n) as an even function of position, whereas the imaginary part can be an odd function of position ($\mathbf{r}$). Therefore, the index profile becomes symmetric with its complex conjugate, $n(\mathbf{r}) = n^*(\mathbf{r})$. Using a pair of coupled waveguides, with one amplifying guided lightwaves while the other dissipating an equal amount, Rüter *et al.* reported the experimental observation of parity-time symmetry in optics[4]. Once the parity-time-symmetry threshold is reached, as shown in Fig. 2a, the optical power in the gain waveguide monotonically increases while it decays in the lossy one. The propagation of light is non-reciprocal in the sense that the power oscillation between the two coupled waveguides is no longer symmetric. However, the transmission of the waveguides remains reciprocal.

Using such a concept, the parity-time optics can be extended into large-scale transversely periodic media, such as photonic lattices and potentially parity-time photonic crystals and metamaterials (Fig. 1). Instead of physically constructing the coupled gain and/or loss waveguide network, Regensburger *et al.* developed a clever approach and emulated the complex dynamics of a large-scale parity-time photonic lattice with a sequence of light pulses propagating in two connected fibre loops[5]. The parity-time symmetry is realized by periodically switching the gain and loss in the two loops while the signal pulses are propagating inside. This unique experimental scheme not only allowed the observation of tailored transverse energy flow in the synthetic medium but also demonstrated the concept of unidirectional invisibility: the entire optical system is invisible when illuminating light from one end but not from the other. However, it is important to note that the unidirectional propagation of light refers only to the

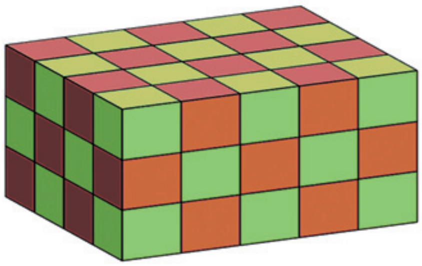

Figure 1 | A photonic crystal consists of periodically arranged non-absorbing dielectrics with sizes at the scale of the light wavelength. Similarly, a parity-time synthetic matter consists of a strong but balanced spatial arrangement of index of refractions, as well as gain and/or loss.

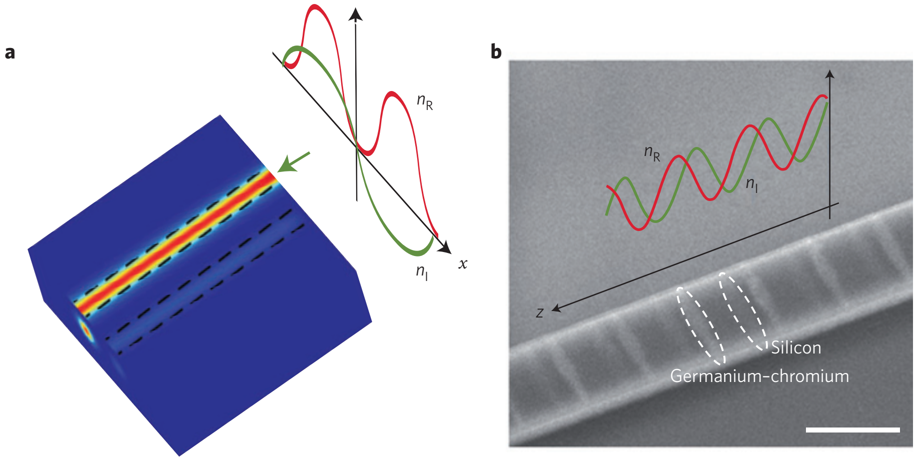

Figure 2 | Unidirectional propagation of light at exceptional points. **a**, A pair of coupled waveguides with parity-time symmetry[4]. The two waveguides are aligned in the transverse direction, one amplifying the guided light while the other dissipating it by an equal amount, allowing for light propagating in only one of the two coupled channels. n_R and n_I are the real and imaginary parts of the indices, respectively. **b**, A scanning electron microscope image of a passive waveguide with periodic modulation in both the index of refraction and the loss parameters along the propagation direction. Pairs of silicon and germanium–chromium pads are deposited on top of a silicon waveguide. Scale bar, 1 μm. The volumes of the silicon and the lossy germanium–chromium pads adjust the modulation depths of the real and imaginary components of the dielectric constants, and control the position of the exceptional points in the parameter space. Panel **b** courtesy of L. Feng.

reflectance. The parity-time symmetry suppresses the reflection from the synthetic medium, making the medium itself invisible; but it does not prevent any other object or scatterer behind the medium from being seen. Although the temporal domain demonstration of the parity-time optic lattices is impressive, the real-space implementation of the parity-time spatial crystal (Fig. 1) is yet to be realized as it demands simultaneous incorporation and very subtle balance of gain and loss at the wavelength scales in composite materials.

> The propagation of light in a parity-time related system is non-reciprocal only in the sense that the power oscillation between the coupled channels is no longer symmetric.

Interestingly, there is a large family of non-Hermitian Hamiltonians that can also produce merged branches of eigen solutions through accidental degeneracy. Thus, the question here is: can these systems yield asymmetric light propagation? Feng *et al.* have recently demonstrated an approach that leads to unidirectional reflectionless propagation of light on a silicon photonic chip[6]. In contrast to the previous idea of using parity-time symmetry, this device utilizes only passive materials without gain media. As shown in Fig. 2b, they employed a simple periodic modulation of the permittivity (ε) of the constituent materials along the propagation direction, $\Delta\varepsilon = \cos(kx) + z\sin(kx)$, with a single complex number z controlling the relative strength and phase between the sinusoidal (odd) and cosinusoidal (even) modulations. Here k and x are the wave vector and the position coordinates, respectively. The scattering matrix in the proposed scheme, however, is non-unitary when modal losses are present, and the parity-time symmetry is therefore always broken. What is remarkable is that the authors still observed unidirectional reflectionless propagation of light in such a passive waveguide — one of the key signatures of a synthetic material with parity-time symmetry. This observation, in fact, originates from the exceptional points that exist in both the passive system and the parity-time synthetic materials possessing active (gain) media.

Being the most important degeneracy intrinsic to non-Hermitian Hamiltonians, the exceptional point is responsible for the experimental observations of level repulsion and crossing, bifurcation and chaos, as well as quantum phase transitions in open systems[18]. Feng *et al.* studied the evolution of the scattering matrix when modal losses are present. Looking at the entire complex plane of z, the relative modulation strength and phase, the eigenvalues of the system are determined by the square-root-type multi-valued Riemann surface, $\sqrt{1+z^2}$. Two exceptional points are located at $z = \pm i$ (Fig. 3) where two branches of solutions meet. At the exceptional points, the sinusoidal and cosinusoidal modulations can be either in or out of phase with equal modulation strengths. More intuitively, the reflectance in both directions is the superposition of the two solutions depicted by the two sheets of the Riemann surface. However, the two solutions coalesce at the exceptional points and the modal interference between the two suppresses the reflectance in one direction but not the other, leading to the unidirectional reflectionless waveguiding without gain media. Even though the parity-time symmetry is always broken, the scattering matrix of the passive system still shares the similar solutions as the parity-time symmetric ones do at the exceptional points. The exceptional points are indeed responsible for the observed unidirectional reflectionless light propagation. Through proper mathematical transformation taking care of the attenuation, the parity-time symmetry can potentially be restored[6,14]. Moreover, the demonstration of such a unidirectional light propagation on a silicon photonic chip may underpin new types of photonic devices for communication and information processing.

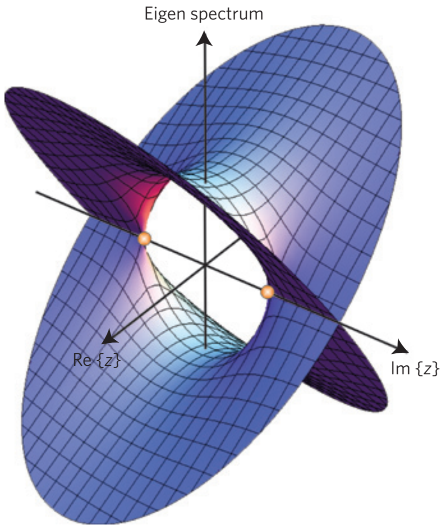

Figure 3 | The two solutions of a two-channel waveguide with a complex and interacting Hamiltonian coalesce at the exceptional points (orange spheres). The destructive interference of the two can lead to the observation of unidirectional reflectionless waveguiding without involving gain media. Re{z} and Im{z} are the real and imaginary parts of the complex parameter, respectively.

The rapidly developing field of plasmonics and metamaterials provides new-fangled opportunities for judicious photonic designs that involve only passive materials.

These intriguing observations originating from the coalescence of states at exceptional points led to a recent debate: whether one is able to break the reciprocity of light and construct an optical diode based on a two-channel waveguide with a spatially varying but time-independent dielectric function[19–21]. To this end, it is important to note that the propagation of light in a parity-time related system is non-reciprocal only in the sense that the power oscillation between the coupled channels is no longer symmetric. The Lorentz-reciprocity of light, as a matter of fact, strictly holds even in these open systems regardless of the presence of loss and/or gain. The present exciting explorations involving the parity-time symmetry or exceptional points do not change the reciprocal nature of the transmitted light in these synthetic media. Such carefully tailored optical materials can be unidirectional in reflection, but do not offer a true Harry Potter magic cloak. Breaking the reciprocity of light by means of magnetic optics and temporal modulations seems indispensable in realizing the fictional cloak of "I can see you, but you cannot see me" — a long awaited breakthrough yet to come. ❐

Xiaobo Yin and Xiang Zhang are in the NSF Nano-scale Science and Engineering Center (NSEC), 3112 Etcheverry Hall, University of California at Berkeley, Berkeley, California 94720, USA, and in the Materials Sciences Division, Lawrence Berkeley National Laboratory, 1 Cyclotron Road, Berkeley, California 94720, USA.
e-mail: xiang@berkeley.edu

References

1. Pendry, J. B., Schurig, D. & Smith, D. R. *Science* **312,** 1780–1782 (2006).
2. Schurig, D. *et al. Science* **314,** 977–980 (2006).
3. Valentine, J., Li, J., Zentgraf, T., Bartal, G. & Zhang, X. *Nature Mater.* **8,** 568–571 (2009).
4. Rüter, C. E. *et al. Nature Phys.* **6,** 192–195 (2010).
5. Regensburger, A. *et al. Nature* **488,** 167–171 (2012).
6. Feng, L. *et al. Nature Mater.* **12,** 108–113 (2013).
7. Yablonovitch, E. *Phys. Rev. Lett.* **58,** 2059–2062 (1987).
8. John, S. *Phys. Rev. Lett.* **58,** 2486–2489 (1987).
9. Strauf, S. *et al. Phys. Rev. Lett.* **96,** 127404 (2006).
10. Raman, A. & Fan, S. *Phys. Rev. Lett.* **104,** 087401 (2010).
11. Musslimani, Z. H., El-Ganainy, R., Makris, K. G. & Christodoulides, D. N. *Phys. Rev. Lett.* **100,** 030402 (2008).
12. Makris, K. G., El-Ganainy, R. & Christodoulides, D. N. *Phys. Rev. Lett.* **100,** 103904 (2008).
13. Mostafazadeh, A. *Phys. Rev. Lett.* **102,** 220402 (2009).
14. Guo, A. *et al. Phys. Rev. Lett.* **103,** 093902 (2009).
15. Bagchia, B. & Quesne, C. *Phys. Lett. A* **273,** 285–292 (2000).
16. Bender, C. M., Brody, D. C. & Jones, H. F. *Phys. Rev. Lett.* **89,** 270401 (2002).
17. Bender, C. M. & Boettcher, S. *Phys. Rev. Lett.* **80,** 5243–5246 (1998).
18. Heiss, W. D. *J. Phys. A* **37,** 2455–2464 (2004).
19. Feng, L. *et al. Science* **333,** 729–733 (2011).
20. Fan, S. H. *et al. Science* **335,** 38 (2012).
21. Feng, L. *et al. Science* **335,** 38 (2012).

Acknowledgements

The authors acknowledge funding support from the US Department of Energy, Office of Basic Energy Sciences under Contract No. DE-AC02-05CH11231 through the Materials Sciences Division of Lawrence Berkeley National Laboratory (LBNL).

Corrected online: 29 April 2013

Correction

In the Commentary 'Unidirectional light propagation at exceptional points' (*Nature Mater.* **12,** 175–177; 2013), the volume number in ref. 6 was incorrect; the reference should have read 'Feng, L. *et al. Nature Mater.* **12,** 108–113 (2013)'. This error has been corrected in the HTML and PDF versions, after print 29 April 2013.